Maximum loading capacities of solid fragments with pores and rough surfaces

Cheng Luo

Department of Mechanical and Aerospace Engineering, University of Texas at Arlington 500 W. First Street, Woolf Hall 226, Arlington, TX 76019, USA
chengluo@uta.edu

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ABSTRACT: Maximum loading capacities of floating solid fragments with pores and rough surfaces have been recently measured and reported by other researchers. The maximum loading capacities were larger than what these researchers expected. They speculated that an extra supporting force was induced by a thick air film, which was generated on outer surfaces of a solid fragment due to surface roughness. They also found that the maximum loading capacities increased with the decrease in the pore sizes. In this work, to interpret these interesting phenomena, we developed a theoretical model, and did force analysis of a configuration that was similar to the reported one. We demonstrated: (i) maximum loading capacities were not necessarily related to pore sizes, while large pores might cause the sinking of the corresponding solid fragment due to the penetration of water into these pores; and (ii) both surface tension and surface roughness contributed to buoyancy through the displacement of additional water, and their degrees of contribution depended on critical sizes of the solid fragment. In addition, we also simplified our model for the cases that the solid fragment was a rectangular plate or a cylinder of rectangular cross sections. Finally, we discussed the case that the solid fragment was a circular disk. © Global Scientific Publishers 2012

KEYWORDS: floatability, surface tension, solid fragments, surface roughness, pores, micro/nanostructures.

1. Introduction

Surface tension is an effect within the surface layer of a liquid. It acts tangentially to the interface between water and a floating object, pointing out of this object. Surface tension allows small objects, such as sewing needles, water striders [1,2], water strider robots [3], mm-scaled boats [4,5], and cm-scaled gels [6], flotillas [7] and boats [8], to float on water surfaces, although these small objects may be more dense than water.

Much work has been done by other researchers to consider floatability of small objects, such as spheres [9], circular disks [10,11], and cylindrical fragments which have circular [9,1-3] or square [9] cross-sections. Surface tension was considered to have significant contribution to buoyant forces of these small objects. On the other hand, it was experimentally demonstrated in [12] that, in case surface tension was negligible, maximum loading capacity of a solid fragment was still larger than buoyant force. The authors of [12] speculated that an extra supporting force was induced by a thick air film, which was generated on outer surfaces of the solid fragment due to the effect of hybrid micro/nanostructures. It was also found in [12] that the maximum loading capacities increased with the decrease in the pore sizes. In this work, to interpret these interesting phenomena, we developed a theoretical model, and did force analysis of a configuration similar to that of [12]. Another motivation to develop this theoretical model was that the solid fragment considered in [12] included two geometric features (i.e., micropores and hybrid micro/nano-structures), whose effects on floatability of a solid fragment have not been previously considered.

2. Theoretical model

2.1 Configuration

The solid fragment considered in this work has a configuration similar to those tested in [12]. It consists of five thin plates (Figs. 1a and 1b). Each plate has a rectangular shape. One of the plates serves as the bottom plate of the fragment. The other four plates are placed vertically on the four edges of this bottom plate, respectively. A cavity is formed inside the solid fragment by these five plates. The authors of [12] speculated that an extra supporting force was induced by a thick air film, which was generated on outer surfaces of the solid fragment due to the effect of hybrid micro/nanostructures. It was also found in [12] that the maximum loading capacities increased with the decrease in the pore sizes. In this work, to interpret these interesting phenomena, we developed a theoretical model, and did force analysis of a configuration similar to that of [12]. Another motivation to develop this theoretical model was that the solid fragment considered in [12] included two geometric features (i.e., micropores and hybrid micro/nano-structures), whose effects on floatability of a solid fragment have not been previously considered.

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the area of the solid part on a unit area of the plate surface is \((1 - f_1)\). Let \(f_2\) be the area that micro/nano-structures occupy on a unit area of this solid portion.

A mass is added in the cavity of the solid fragment. Let \(w\) represent its weight. If \(w\) is large enough, the fragment may sink into water. Figs. 1(a) and 1(b) give a configuration before the solid fragment is submerged in water. This solid fragment is located below the original water surface. A curved air/water interface is formed beside the fragment. This surface extends from the original water surface to the top surface of the solid fragment. Let \(h_1\) denote the height of the air/water interface. Each of the four vertical plates can be considered as a long strip, since its thickness is much less than another two dimensions. Accordingly, the profile of the air/water interface on every vertical plate has a cylindrical shape. As illustrated in Figs. 1(a) and 1(c), let \(AB\) denote the cross-sectional profile of the cylindrical interface on a vertical plate, where \(A\) represents the three-phase (air/liquid/solid) contact line located at the top surface of the vertical plate and \(B\) is the intersection of the air/water interface with the original water surface. Set \(\theta_c\) to be the intrinsic contact angle of water on a plate. Let \(\theta_0\) denote the angle subtended by the horizontal plane and the tangent to \(AB\) at \(A\). The top corners of a vertical plate are considered to be round, whose radii are much smaller than the three dimensions of the plate. When the weight of the mass is gradually increased, the contact line \(A\) moves on the top surface of the plate from the outer edge to the inner edge (Fig. 1d). During this moving process, \(\theta_0\) changes from \((\theta_c - 90^\circ)\) to \((\theta_c + 90^\circ)\) [13]. After \(A\) passes the inner edge and gets into the cavity of the solid fragment, water fills the cavity, making the fragment sink [13]. In addition,
the fragment may sink due to the penetration of water into pores, which is another way that water may fill the cavity.

2.2 Supporting force and physical meaning

Assume that water neither penetrates into the pores nor goes into the valleys of the hybrid structures. This assumption will be examined in Sub-section 2.5. Consider an object that comprises the fragment, the cavity inside the fragment, hybrid micro/nanostructures, and air trapped in the valleys between these micro/nanostructures. According to free-body diagram given in Fig. 1(e), this object suffers air pressure, water pressure, surface-tension induced force, and weights of the solid fragment, hybrid structures and added mass. The air pressure equals atmospheric pressure, and is denoted by \( p_0 \). Let \( F_s \) denote the total supporting force. This force is the vertical resultant of air pressure, water pressure and surface-tension-induced force. In reference to Fig. 1(e), \( F_s \) is calculated to be

\[
F_s = \rho_w gV_1 + \rho_w gV_2 + \rho_w gV_3 + 2\gamma \sin \theta_0 (a + b) \tag{1}
\]

where \( \rho_w \) denotes water density, \( g \) is gravitational acceleration, and \( \gamma \) represents surface tension of water. In Eq. (1),

\[
V_i = abc, \ V_2 = abh_1, \ V_3 = h_2(1 - f_1)(ab + 2bc + 2ac) \tag{2}
\]

where \( h_2 \) denotes the height of the hybrid micro/nanostructures. As marked in Fig. 1(a), \( V_1 \) is actually the total volume of the solid fragment and the cavity inside this fragment, \( V_2 \) is the volume of the region enclosed by vertical surfaces, original water surface, and top surface of the solid fragment, and \( V_3 \) is the total volume of the hybrid structures and the valleys between these structures.

Next, we explore the geometric meaning of the fourth term on the right-hand side of Eq. (1). Consider force balance on the air/water interface (Fig. 1f). On the original water surface, \( \gamma \) is along the horizontal direction. Let \( h \) denote the vertical distance between the original water surface and a point on the air/water interface. Then air and water pressures at this point are \( p_0 \) and \( (p_0 + \rho_w gh) \), respectively. It is readily shown that the vertical force generated by the air and water pressures has a magnitude of \( \rho_w gV_4 \), where \( V_4 \) is the volume formed by the air/water profile, the original water surface and the vertical surface. Accordingly, based on the balance of forces along the vertical direction (Fig. 1f), we have

\[
2\gamma \sin \theta_0 (a + b) = \rho_w gV_4 \tag{3}
\]

It follows from this equation that

\[
V_4 = C \sin \theta_0 (a + b) \tag{4}
\]

where \( C = 2\gamma / \rho_w g \) and equals \( 1.49 \times 10^{-4} \text{ m}^2 \) \( (\gamma = 72.7 \text{ mN/m}, \ g = 9.8 \text{ N/kg} \) and \( \rho_w = 10^3 \text{ kg/m}^3 \) are used in this calculation). This equation implies that the maximum value of \( V_4 \) is \( C(a + b) \), which corresponds to \( \theta_0 = 90^\circ \) (at this angle, as illustrated in Fig. 1d, \( A \) is located on the outer or inner edge of the vertical plate if \( \theta > 90^\circ \) or \( \theta _c < 90^\circ \)). Set

\[
V = V_1 + V_2 + V_3 + V_4 \tag{5}
\]

Then, with the aid of Eqs. (3) and (5), it follows from Eq. (1) that

\[
F_s = \rho_w g(V_1 + V_2 + V_3 + V_4) = \rho_w gV \tag{6}
\]

As observed from Fig. 1(a), \( V \) is actually the total volume of water displaced due to the floating of the solid fragment on water surface. Therefore, \( F_s \) equals the total weight of the displaced water, and is the buoyancy that the solid fragment has.

2.3 Height of the air/water interface, and effect of surface tension on buoyancy

To have a better understanding about \( V_2 \) (whose expression is given in Eq. (2)), we determine the relation between \( h_1 \) and \( \theta_0 \). Set up a rectangular coordinate system \((x, y, z)\) (Fig. 1c). The origin is at \( A \), and \( x \)- and \( y \)-axes are along the horizontal and vertical directions, respectively. The air/water interface can be determined by solving Young-Laplace equation [14]:

\[
\gamma \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \rho_w g(h_1 - y) \tag{7}
\]

where \( R_1 \) and \( R_2 \) are, respectively, radii of the two principal curvatures at a point of the air/water interface. The mean curvature is one half of \((1/R_1 + 1/R_2)\). \( R_1 \) and \( R_2 \) are considered positive if their associated curves on the air/water surface bend towards air. The two principal curvatures at a point of \( AB \) are, respectively, 0 and \( 1/R \), where \( R \) denotes the radius of curvature of \( AB \). Hence, at a point on \( AB \),

\[
\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R} \tag{8}
\]

By Eqs. (7) and (8), we have

\[
\frac{1}{R} = \frac{\rho_w g}{\gamma} (h_1 - y) \tag{9}
\]

Let \( \theta(x) \) denote the angle between the horizontal plane and the tangent to \( AB \). Then,

\[
\frac{1}{R} = -\frac{d\theta(x)}{ds} \tag{10}
\]

where \( s \) denotes arc length of \( AB \). By Eqs. (9) and (10), we have

\[
-\frac{d\theta(x)}{ds} = \frac{\rho_w g}{\gamma} (h_1 - y) \tag{11}
\]

It is observed from this equation that
$$\frac{d\theta(0)}{ds} = -\rho_g g h_1, \quad \left. \frac{d\theta(x)}{ds} \right|_{x=\infty} = 0 \tag{12}$$

In addition, there are two additional boundary conditions for Eq. (11):

$$\theta(0) = \theta_0, \quad \left. \theta(x) \right|_{x=\infty} = 0 \tag{13}$$

With the aid of Eqs. (12) and (13), the differentiation of Eq. (11) with respect to $s$, followed by the integration with respect to $\theta(x)$, leads to

$$h_1 = 2 \sin \frac{\theta_0}{2} \sqrt{\frac{\gamma}{\rho_g g}} \tag{14}$$

Similar relations have been previously derived by other researchers (see, for example, Eq. (74) in Chapter 6 of [15]). In Eq. (14), $\sqrt{\gamma/\rho_g g}$ represents capillary length of water, and equals 2.73 mm. As discussed in Subsection 2.1, the range of $\theta_0$ is related to the value of $\theta_1$.

When $\theta_1 > 90^\circ$, the maximum value of $h_1$ is 5.46 mm, which corresponds to the case $\theta_0 = 180^\circ$ (at this angle, $A$ is located on the inner edge of the vertical plate as illustrated in Fig. 1d). Otherwise, the maximum value of $h_1$ is $5.46 \sin(\theta_1/2 + \pi/4)$ mm, which corresponds to that $\theta_0 = \theta_1 + \pi/2$ (at this angle, $A$ is also located on the inner edge of the vertical plate). In either case, $h_1$ has the maximum order of 1 mm.

With the aid of Eqs. (1), (2) and (14), it is readily shown that

$$F_s = \rho_g g abc + \rho_w g (1-f_1)h_2 (ab+2bc+2ac)$$

$$+ 2ab\sqrt{\rho_w g \gamma} \sin \frac{\theta_0}{2} + 2 \gamma (a+b) \sin \theta_0 \tag{15}$$

It can be observed from this equation that four supporting force contribute to the buoyancy. The last two supporting forces (represented by the third and fourth terms on the right-hand side of the equation, respectively) are both tension-related forces. An approach that is commonly applied to examine the degree that each supporting force contributes to the buoyancy is to compare critical dimensions of the solid fragment with capillary length of water [1], which is 2.73 mm. Using this approach, the expression of $F_s$ can be simplified in the following two cases.

1. When the critical dimensions are much larger than 2.73 mm, the two surface tension-related forces are negligible in comparison with $\rho_w g abc$. For example, when $a$, $b$ and $c$ have the same order of 0.1 m or above, $\rho_w g abc$ is much larger than the two surface tension-related forces. Also, since $h_2$ normally has an order below 1 mm, $\rho_w g (1-f_1)h_2 \times (ab+2bc+2ac)$ can also be neglected in comparison with $\rho_w g abc$. Accordingly, in practice, the buoyancy of a large-scale object is considered to be $\rho_w g abc$.

2. If the critical dimensions are much smaller than 2.73 mm, then both $\rho_w g abc$ and $2ab\sqrt{\rho_w g \gamma} \sin \theta_0 / 2$ are negligible in comparison with $2 \gamma (a+b) \sin \theta_0$. This is the case, for example, when $a$, $b$ and $c$ have the same order of 10 µm or below. Likewise, since $h_2$ may be comparable with $a$, $b$ and $c$, $\rho_w g (1-f_1)h_2 (ab+2bc+2ac)$ should be neglected as well. Thus, in this case, the buoyancy is considered to equal $2 \gamma (a+b) \sin \theta_0$. In [1], the legs of water striders were modeled as circular cylinders with a small radius, which equalled 40 µm and was much smaller than 2.73 mm. Accordingly, in [1], the buoyancy per unit length along the leg direction was calculated to be $2 \gamma \sin \theta_0$.

On the other hand, when the critical dimensions of a solid fragment are neither much larger nor much smaller than 2.73 mm, all the four supporting forces may have significant contribution to the buoyancy.

2.4 Maximum loading capacity

Let $F_s_{\text{max}}$ denote the maximum value of $F_s$. With the aid of Eqs. (1), (2) and (15), it is readily shown that

$$F_s_{\text{max}} = \rho_w g abc + \rho_w g (1-f_1)h_2 (ab+2bc+2ac)$$

$$+ 2ab\sqrt{\rho_w g \gamma} \sin \frac{\theta_{0\text{max}}}{2} + 2 \gamma (a+b) \sin \theta_{0\text{max}} \tag{16}$$

where $\theta_{0\text{max}}$ is the solution to the equation

$$ab\sqrt{\rho_w g \gamma} \cos \frac{\theta_0}{2} + 2 \gamma (a+b) \cos \theta_0 = 0 \tag{17}$$

Considering the balance of forces along the vertical direction (Fig. 1e), we have

$$w = F_s - \rho_s g(1-f_1)(ab+2bc+2ac)$$

$$- \rho_h g f_2 (1-f_1)(ab+2bc+2ac) \tag{18}$$

where $\rho_s$ and $\rho_h$ denote densities of the solid fragment and hybrid structures, respectively. The second and third terms on the right-hand side of this equation represent the weights of the solid fragment and the hybrid structures, separately. Let $w_{\text{max}}$ denote the maximum weight of the added mass that still allows the solid fragment to float on water surface. It is also the maximum loading capacity of this fragment. In terms of Eq. (18), we have

$$w_{\text{max}} = F_s_{\text{max}}$$

$$- g(1-f_1)(\rho_h f_2 h_2 + \rho_s f_1)(ab+2bc+2ac) \tag{19}$$

By Eqs. (16) and (19), we get
\[ w_{\text{max}} = \rho_{\text{gabc}} + 2ab\sqrt{\rho_{s}g\gamma} \sin \frac{\theta_{\text{max}}}{2} + 2b\sqrt{\rho_{s}g\gamma} \sin \frac{\theta_{\text{max}}}{2} + 2\gamma(a+b) \sin \theta_{\text{max}} \]

(20)

It is observed from this equation that only the last term on the right-hand side is related to the pore sizes and hybrid micro/nanostructures. If \( h_{2} \) and \( t \) are both much smaller than the three critical dimensions \( a, b \), and \( c \), then the last term is negligible in comparison with \( \rho_{s}gabc \). Accordingly, in this case, \( w_{\text{max}} \) is independent of both pore sizes and hybrid micro/nanostructures. In case \( h_{2} \) and \( t \) are comparable to the three critical dimensions, \( w_{\text{max}} \) is affected by pore sizes and hybrid micro/nanostructures. According to these discussions, whether \( w_{\text{max}} \) is related to pores and hybrid micro/nanostructures depends on both their sizes and the critical dimensions of the solid fragment.

### 2.5 Penetration of water into hybrid micro/nanostructures and pores

There exist two possible wetting states on a micro/nanostructured solid surface: Wenzel [16] and Cassie–Baxter [17]. In the Wenzel state, water completely penetrates between two micro/nanostructures (e.g., lines and pillars), while in the Cassie–Baxter state air is trapped between these micro/nanostructures and water stays on top of the micro/nanostructures and trapped air [18-23]. The Cassie–Baxter and Wenzel may co-exist on a micro/nanostructure-formed surface [24]. In the Cassie-Baxter state, the thickness of the trap air equals the height of the micro/nanostructures.

In case water pressure is not high, air may be trapped inside the valleys between hybrid micro/nanostructures and consequently the wetting is in the Cassie-Baxter state. Also, water may not penetrate the pores and an air/water interface may exist in each pore. On the other hand, when the solid fragment is deep inside water, due to high water pressure, water is easy to penetrate into the valley or pore. Let \( e \) denote the largest distance between two adjacent hybrid micro/nanostructures (Fig. 2a). Set \( l \) to be the largest lateral dimension of the pore.

As illustrated in Figs. 2(a) and 2(b), let \( A_{1}B_{1}C_{1} \) denote a cross-sectional profile of the air/water interface suspended on or inside a pore. Without loss of generalization, the following analysis focuses on the three-phase (air/liquid/solid) contact point \( A_{1} \) located at one end of \( A_{1}B_{1}C_{1} \). The same analysis also applies to the other contact point \( C_{1} \). Set \( \theta_{l} \) to be the angle subtended by the vertical direction and the tangent to \( A_{1}B_{1}C_{1} \) at \( A_{1} \). Both top and bottom corners of a pore are considered to be round, whose radii are much smaller than the width and height of the pore. With the increase in the water depth (i.e., the increase in water pressure), the contact point \( A_{1} \) moves from the bottom corner to the top corner of the pore through the pore sidewall (Fig. 2b). During this moving process, \( \theta_{l} \) changes from \((270^\circ - \theta_{e})\) to \((90^\circ - \theta_{e})\) [13, 20]. After \( A_{1} \) passes the top corner and gets into the cavity of the solid fragment, water fills the cavity, making the fragment sink [13].

The water pressure, \( p_{w} \), is related to \( \theta_{l} \) by [25]

\[ p_{w} = \frac{2\gamma \cos \theta_{l}}{l} + p_{0} \]

(21)

As \( \theta_{l} \) changes from \((270^\circ - \theta_{e})\) to \((90^\circ - \theta_{e})\), \( p_{w} \) varies between \((-2\gamma \sin \theta_{l} / l + p_{0})\) and \((2\gamma \sin \theta_{l} / l + p_{0})\). Let \( p_{w_{\text{max}}} \) denote the critical water pressure that is needed for water to penetrate a pore and fill the cavity. When \( \theta_{l} \) is equal to or larger than \( 90^\circ \) (i.e., the pore surface is not hydrophilic),

\[ p_{w_{\text{max}}} = \frac{2\gamma}{l} + p_{0} \]

(22)

If \( \theta_{e} \) is less than \( 90^\circ \) (i.e., the pore surface is hydrophilic), \( p_{w_{\text{max}}} = 2\gamma \sin \theta_{l} / l + p_{0} \). Three points can be observed from these two maximum values. First, the maximum water pressure needed for water to penetrate a hydrophobic pore is larger than that required to penetrate a hydrophilic pore. Second, the maximum water pressure needed for water to penetrate a hydrophilic pore increases with
the intrinsic contact angle, while this pressure is constant when the pore has a hydrophobic surface. Third, the maximum water pressure required for penetration decreases with the increase in the pore size, and consequently water is easy to penetrate large pores, making the corresponding fragment sink.

Following the same line of reasoning, the critical water pressure for water to get into the valley between two hybrid micro/nanostructures, $P_{\text{w max}}$, is

$$P_{\text{w max}} = \frac{-2\gamma \cos \theta}{e} + p_0 \quad (23)$$

This equation holds for $\theta > 90^\circ$. Otherwise, $P_{\text{w max}}$ is $P_{\text{w max}} = p_0$. That is, once water contacts hybrid micro/nanostructures that have hydrophilic surfaces, it gets into the valley.

Next, we focus on the case that the surfaces of pores and hybrid micro/nanostructures are hydrophobic. The same analysis applies to the situation that these surfaces are hydrophobic. Use $d$ to denote the height difference between the bottom of solid fragment and the original water surface. Then, as illustrated in Fig. 1(a),

$$d = c + h$$

By Eqs. (22)-(24), the critical values of $d$ that result in the water penetration are:

$$d_{cr1} = \frac{-2\gamma \cos \theta}{e \rho wg}, \quad d_{cr2} = \frac{2\gamma}{l \rho wg}$$

where $d_{cr1}$ and $d_{cr2}$ denote the critical values of $d$ for the valley and pore, respectively. The relations in Eq. (25) indicate that $d_{cr1}$ and $d_{cr2}$ decrease with the increase in $e$ and $l$, respectively. To avoid that water gets into the valleys or penetrates the pores, the valleys and pores should be designed to have small sizes.

### 2.6 Comparison with experimental results of [12]

Next, let’s consider solid fragments of [12], in which $a = 4$ cm, $b = 2$ cm, and $c = 1$ cm. According to images of scanning electron microscopy shown in [12], the height of hybrid micro/nanostructures was less than 10 μm, i.e., $h_2 < 10$ μm. The total weights of the solid fragments and the hybrid micro/nanostructures range from 0.004 N to 0.006 N. The value of $\theta_0$ was measured to be 156°, which means that the plate surface was super-hydrophobic. According to the discussion in Sec. 2.1, $\theta_0$ ranges between 66° and 246°. By Eq. (17), $\theta_{\text{max}} = 180°$.

The comparisons of $\rho_w gabc$ with other terms on the right-hand side of Eq. (20) indicate that only $\rho_w gabc$ and $2ab\sqrt{\rho_w g} \sin(\theta_{\text{max}}/2)$ have significant contribution to $w_{\text{max}}$. Accordingly, by Eq. (20), we have

$$w_{\text{max}} = \rho_w gabc + 2ab\sqrt{\rho_w g} = 1.19 \text{ N} \quad (26)$$

As observed from this equation, $w_{\text{max}}$ is in fact independent of pore sizes. In [12], maximum loading capacities were experimentally measured for eight fragments whose pore sizes ranged from 0 to 930 μm. According to our calculation, the average of these maximum loading capacities is 1.25 N, which is close to our theoretically predicted value of 1.19 N. It was indicated in [12] that the eight measured capacities increased with the decrease in the pore sizes. On the other hand, we find that the differences of these capacities with their average value are less than 0.10 N. Accordingly, they may be considered to be independent of pore sizes.

As indicated in [12], the contribution of $2\gamma(a+b)\sin \theta_{\text{max}}$ to $w_{\text{max}}$ is negligible. However, in [12], the contribution of $2ab\sqrt{\rho_w g} \sin(\theta_{\text{max}}/2)$ to $w_{\text{max}}$ was not considered, and $w_{\text{max}}$ was speculated to be the summation of $\rho_w gabc$ and $\rho_w g(1-f_1)h_2 (ab+2bc+2ac)$. To make this speculation hold, $h_2$ was assumed to be 1.5 mm or larger. This assumption actually does not hold, since $h_2 < 10$ μm. Consequently, $\rho_w g(1-f_1)h_2 \times(ab+2bc+2ac)$ has negligible effect on $w_{\text{max}}$. In Eq.

Figure 3. Schematics of (a) a rectangular fragment, (b) a cylinder with rectangular cross-sections, and (c) a circular disk.
(26), \( \rho_{wabc} = 0.77 \) N, and \( 2ab\sqrt{\rho_{w}g} \gamma \) = 0.42 N. These values indicate that \( 2ab\sqrt{\rho_{w}g} \gamma \) has significant contribution to \( w_{\text{max}} \), making \( w_{\text{max}} \) much larger than \( \rho_{wabc} \). 2ab\sqrt{\rho_{w}g} \gamma \) is a surface tension-induced force, while \( \rho_{s}g(1-f_{i})h_{i}(ab+2bc+2ac) \) is a surface roughness-induced force. Accordingly, surface tension, instead of surface roughness, has much effect on \( w_{\text{max}} \) in the case of [12].

In [12], the type of solid fragments with the largest pore size of 930 \( \mu \) has the smallest value of \( d_{\text{r2}} \). According to Eq. (25), this value is calculated to be 1.60 cm. By Eqs. (14) and (24), the largest possible value of \( d \) for this type of solid fragments is 1.55 cm, less than 1.60 cm. Hence, water should not penetrate into pores during the loading tests of all the solid fragments. That is, the sinking of a solid fragment is only induced by the maximum loading, which is actually the case in [12].

3. Three fragments with simpler geometric shapes

In this section, we consider three solid fragments whose geometric shapes are simpler than the one shown in Fig. 1: rectangular plates, cylinders with rectangular cross-sections, and circular disks (Fig. 3).

3.1 Rectangular plates, and cylinders with rectangular cross-sections

Eq. (20) can be simplified for both rectangular plates and cylinders with rectangular cross-sections. In the case of a rectangular plate, after setting \( c \) to be \( t \), it follows from Eq. (20) that

\[
w_{\text{max}} = \rho_{w}gabt + 2ab\sqrt{\rho_{w}g} \gamma \sin \theta_{\text{max}}^{0} \frac{\theta_{\text{max}}}{2}
+ 2\gamma(a+b)\sin \theta_{\text{max}}^{0}
\]

(27)

As shown in Fig. 3(a), if this rectangular fragment has neither pore structures nor micro/nano features (i.e., \( h_{2} = f_{1} = 0 \)) , then Eq. (27) is reduced to

\[
w_{\text{max}} = (\rho_{w} - \rho_{s})gabt + 2ab\sqrt{\rho_{w}g} \gamma \sin \theta_{\text{max}}^{0} \frac{\theta_{\text{max}}}{2}
+ 2\gamma(a+b)\sin \theta_{\text{max}}^{0}
\]

(28)

Consider a cylinder with rectangular cross-sections. It floats on a water surface with its longitudinal axis parallel to the water surface. Suppose that this fragment also has neither pore structures nor micro/nano features. Let \( \bar{w}_{\text{max}} \) denote its maximum loading capacity per unit length along the cylindrical direction. Set \( a \) and \( t \) to be the width and height of this cylinder, and use \( b \) to represent its length (Fig. 3b). Accordingly, \( b \) is much larger than both \( a \) and \( t \). Subsequently, it follows from Eq. (28) that

\[
\bar{w}_{\text{max}} = (\rho_{w} - \rho_{s})gabt
+ 2a\sqrt{\rho_{w}g} \gamma \sin \theta_{\text{max}}^{0} \frac{\theta_{\text{max}}}{2}
\]

(29)

In case \( t \) equals \( a \), i.e., the cylinder has square cross-sections, the corresponding maximum loading capacity is

\[
\bar{w}_{\text{max}} = (\rho_{w} - \rho_{s})ga^{2}
+ 2a\sqrt{\rho_{w}g} \gamma \sin \theta_{\text{max}}^{0} \frac{\theta_{\text{max}}}{2}
\]

(30)

which is equivalent to Eq. (91) of [9], which is equilibrium condition when a cylinder with square cross-sections floats on a liquid surface.

### 3.2 Circular disk

The floatability of a circular disk on a liquid surface was considered in [11]. Based on the result of [11], the total supporting force when water reaches the top edge of the circular disk is

\[
F_{w} = \rho_{w}g\pi r^{2}t + \rho_{w}g\pi r^{2}h_{j} + 2\gamma \pi r \sin \theta_{0}
\]

(31)

where \( r \) denotes the radius of the disk (Fig. 3c), and \( h_{j} \) represents the height difference between the top surface of the disk and the original water surface and is related to \( r \) and \( \theta_{0} \) as

\[
h_{j} = \left[ \frac{2\gamma}{\rho_{w}g} \left( 1 - r \cos \theta_{0} \right) \right]^{1/2} \left[ 1 + \left( \frac{\gamma}{\rho_{w}g} \right)^{1/2} \frac{1}{r} \right]^{1/2}
\]

(32)

Eq. (32) is an approximate expression of meniscus height. It was originally derived in [26] by the authors of [11] based on the assumption that the meniscus was narrow relative to the diameter of the circular disk. Therefore, it is expected that Eq. (32) gives good prediction of meniscus height only when \( r > L \), where \( L \) represents capillary length of water (2.73 mm). For the case \( r \leq L \), the following expression may be adopted to approximate meniscus height:

\[
h_{j} = r \cos \theta_{0} \left[ \ln \left( \frac{4L}{r(1 + \sin \theta_{0})} \right) - 0.577 \right]
\]

(33)

This equation was derived in [27] under the assumption that \( r \) was small in comparison with \( L \). It was shown in [27] that, when \( 0 < r < 0.07L \), Eq. (33) has accuracy to 1% in comparison with numerical solution to the governing equation of a meniscus profile (i.e., Young-Laplace equation). However, the error of Eq. (33) increases with the increase in the value of \( r \), and is about 10% when \( r = L \) [27].

For a given circular disk, the corresponding maximum loading capacity can be readily determined using Eqs. (31)-(33). In addition, equilibrium conditions for spheres and cylinders with circular cross-sections to float on liquid surfaces have been previously considered by other researchers (see, for example, [9]). Using these equilibrium conditions, the corresponding buoyant forces and maximum loading capacities of the two types of objects can also be easily derived.
4. Summary and conclusions

In this work, we developed a theoretical model to consider the floatability of a solid fragment with pores and rough surfaces. We demonstrated that both surface tension and surface roughness might contribute to the increase in the buoyancy through the displacement of additional water, and that their degrees of contribution depended on critical sizes of the solid fragment. In case the weight of a solid fragment is much smaller than the buoyancy, the creation of the pores does not affect the maximum loading capacity of the solid fragment. Otherwise, the effect of the pore sizes should be considered. In the meanwhile, water might be easy to penetrate large pores, causing a solid fragment to sink. Based on the theoretical model, we further interpreted some interesting experimental phenomena observed in [12]. We also simplified our model for the cases that the solid fragment was a rectangular plate or a cylinder with rectangular cross sections. Finally, we discussed the case that the solid fragment was a circular disk.

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